Mathematical Formulas Describing the Sequences of the Periodic Table

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ABSTRACT: Mathematical formulas describing all of the sequences of the chemical elements are derived from double tetrahedron face-centered cubic lattice model.

Key words: Periodic table; integer sequences; analytical solution; atomic quantum numbers; symmetry; nuclear structure

Introduction

The modern version of the periodic table was developed by Mendeleyev [1, 2] in 1869. The original table relied on the detected relationship between the properties of the elements and the atomic weights of the elements. Anton van den Broek suggested that the fundamental organizing principle of the table is not the weight but rather the nuclear charge [3, 4]. Charge distribution of the nucleus affects the electron density distribution of the atoms [5], thus the sequence of the nuclear charge distribution might show resemblances to the periodicity of the elements. Investigating the periodicity of the nuclear charge occurring in the structural development of double tetrahedron nucleus reveals the periodicity of the elements. The analytical solution describing this periodicity is derived here.

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Sequences of the Periodic Table

There are three sequences in the periodic table [Fig. 1(e)]. Following the first period, each of the periods is repeated in the periodic table. This general or fundamental sequence is described as:

$$S_{\text{fundamental}} = \{1, 2, 2, 3, 3, 4, 4, \ \ldots\}. \tag{1}$$

I will call the numbers in Eq. (1) the sequence numbers of the periods. The number of elements within a period [$\Delta Z(n)$] has the sequence of

$$S_{\Delta z} = \{2, 8, 8, 18, 18, 32, 32 \ldots\}. \tag{2}$$

The atomic number or the nuclear charge of the elements [$Z(n)$] in a completely developed period follows the sequence

$$S_z = \{2, 10, 18, 36, 54, 86, 118 \ldots\}. \tag{3}$$

These sequences bear physical meaning only for the known periods and their extension is only numerical. I will call the sequences in Eqs. (1)–(3) fundamental, periodic, and atomic number sequences of the periodic table, respectively. The formula only for the periodic sequence [$\Delta Z(n)$] is known [8, 9] and the number of elements in a period is determined as:

$$\Delta Z(n) = \frac{1}{8}\{[2n + 3 + (-1)^n]^2\} \tag{4}$$

where n is the number of the period. Recently, Kryachko suggested a new formula [10] for the periodic sequence:

$$\Delta Z(n) = 2\left\{\left[\frac{n}{2} + \frac{(-1)^n - 1}{4}\right] + 1\right\}^2. \tag{5}$$

No numerical solutions describing the fundamental and the atomic number sequences of the periodic table are known [11].

Double Tetrahedron Nuclear Structure

A double tetrahedron shape with alternately arranged protons and neutrons in face-centered cubic lattice has been proposed for the structure of the nucleus [12]. This structure reproduces the symmetry of both quantum mechanics and the periodic system with no discrepancy. The structure is developed from a core tetrahedron (four nucleons) by expanding one extra layer at each period. The number of charges in the outer shell and in the nucleus are identical with the periodicity of the elements in the periodic system (Fig. 1). Deriving analytical solutions for the number of charges in the shell and the nucleus describes the sequences of the periodic table.

Analytical Description of the Sequences

FUNDAMENTAL SEQUENCE

The relationship between the periods (n) and the sequence numbers (m) can be described as:

$$m = \frac{2n + (-1)^n + 3}{4} \quad \text{where } n \in N^*. \tag{6}$$

PERIODIC SEQUENCE

The number of nucleons in the kth layer of a tetrahedron can be calculated by the triangular number [$\mathrm{Tr}(k)$] [3, 4] [Fig. 1(a)]

$$\mathrm{Tr}(k) = \frac{k}{2}(k + 1). \tag{7}$$

In each structural step of its development, the tetrahedron is expanded by one layer in two directions [Fig. 1(b,c)] giving the relationship between the tetrahedron layers and the sequence numbers as:

$$k = 2m. \tag{8}$$

The number of nucleons in the outer shell of the tetrahedron [$\mathrm{Tr}(m)$] is the sum of the two consecutive triangular numbers.

$$\mathrm{Tr}(m) = \mathrm{Tr}(k) + \mathrm{Tr}(k-1) = \mathrm{Tr}(2m) + \mathrm{Tr}(2m-1) = 4m^2 \tag{9}$$

The number of charges in the completely developed shell is

$$\Delta Z(n) = \frac{\mathrm{Tr}(m)}{2} = 2m^2. \tag{10}$$

Substituting the sequence number from Eq. (6) recovers the Tomkeieff formula Eq. (4).

$$\Delta Z(n) = \frac{1}{8}[2n + (-1)^n + 3]^2 \tag{11}$$

ATOMIC NUMBER SEQUENCE

A formula giving the total number of charges in the nucleus with completely developed shells can be derived in a similar manner. The total number of nucleons in a tetrahedron with k layers can be determined by its tetrahedral number [$\mathrm{Th}(k)$] [13, 15]

$$\mathrm{Th}(k) = \frac{k}{6}(k + 1)(k + 2). \tag{12}$$

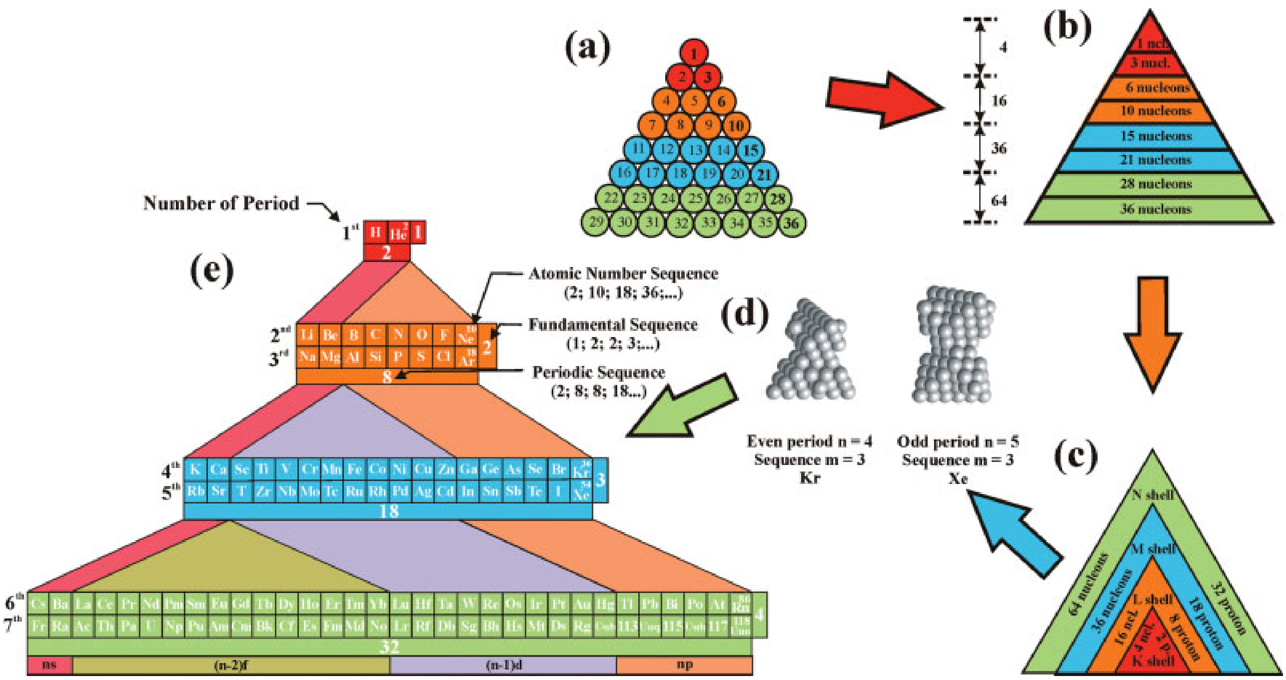

FIGURE 1. The sequences of the periodic table and their reproduction by the charge distribution of the double tetrahedrons nuclear structure. (**a**). The number of nucleons in the triangles. (**b**) Forming a tetrahedron by stacking triangular layers. The nucleons are arranged in a face-centered cubic lattice. (**c**) The number of nucleons and protons in the shells of the fully developed tetrahedron. (**d**) Examples of completely developed double tetrahedron nucleuses are shown for even and odd periods. The 3D images of Kr and Xe. (**e**) The sequences of the double tetrahedron nuclear lattice structure are identical with the quantum mechanical predictions and the periodic table. Pyramid arrangement of the elements in the periodic system [6] as developed by William B Jensen. The periods are numbered at the left while the sequence numbers (explained in the text) are given on the right. Elements 112, 114, 116 and 118 have been reported, but not fully authenticated and the assigned names are only provisional [7]. Elements 113, 115, and 117 have yet to be discovered. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

Substituting the sequence number from Eq. (8) gives the number of nucleons in a tetrahedron for sequence (m) as:

$$\mathrm{Th}(m) = \frac{m}{3}(2m+1)(2m+2) = \frac{4m^3}{3} + 2m^2 + \frac{2m}{3}. \quad (13)$$

The double tetrahedron nucleus is developed by alternately expanding the tetrahedrons [Fig. 1(d)]. The number of nucleons in the double tetrahedron is

$$\mathrm{Th}^{\mathrm{double}}(n) = 2\mathrm{Th}(m) - \mathrm{Tr}^{\text{even-period}}(m) - 4. \quad (14)$$

The formula

$$\frac{(-1)^n + 1}{2} \quad (15)$$

can be used to generate 0 for odd periods and 1 for even number periods, and

$$\mathrm{Tr}^{\text{even-period}}(m) = \frac{(-1)^n + 1}{2}\mathrm{Tr}(m). \quad (16)$$

Equation (14) can be rewritten then as

$$\mathrm{Th}^{\mathrm{double}}(n) = 2\mathrm{Th}(m) - \frac{(-1)^n + 1}{2}\mathrm{Tr}(m) - 4. \quad (17)$$

The number of charges in the nucleus in a completely developed sequence is

$$Z(n) = \frac{\mathrm{Th}^{\mathrm{double}}(n)}{2}. \quad (18)$$

Combining Eqs. (6), (9), (13), (14), (17), and (18) gives the number of nuclear charges for any period. The atomic number sequence of the periodic table can be described then as:

$$Z(n) = \frac{1}{3}\{4m^3 - [(-1)^n - 1]3m^2 + 2m - 6\}. \quad (19)$$

Substituting m from Eq. (6) gives the atomic number sequence

$$Z(n) = \frac{1}{48}[2n + (-1)^n + 3]^3 - \frac{1}{16}[(-1)^n - 1][2n + (-1)^n + 3]^2 + \frac{1}{6}[2n + (-1)^n + 3] - 2. \quad (20)$$

Conclusion

The symmetry equivalence between the charge distribution of fcc double tetrahedron lattice and the periodicity of the elements is used to derive analytical solutions for the sequences present in the periodic table. The derived Eqs. (6), (11), and (20) reproduce the fundamental, periodic, and atomic number sequences of the periodic table, respectively.

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